

Six Simultaneous Equations With Integer Solution

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ABSTRACT

In the below paper we have attempted to find integer solution to six simultaneous equations. In math literature we have not found a similar equation that has been investigated before. The equations consist of three quadratics on the left hand side which sum up to a square on the right side. Even-though the equations are quadratic in nature the degree of difficulty increases with the number of equations considered. The author considered six equations simultaneously. Others can try & solve simultaneous equation which are more than six in the group.

Consider the below equations:

$$(p)x + (q)y + (r)z = a^2 \text{ --- (1)}$$

$$(p + 1)x + (q + 1)y + (r + 1)z = b^2 \text{ --- (2)}$$

$$(p + 2)x + (q + 2)y + (r + 2)z = c^2 \text{ --- (3)}$$

$$(p + 5)x + (q + 5)y + (r + 5)z = d^2 \text{ --- (4)}$$

$$(p + 7)x + (q + 7)y + (r + 7)z = e^2 \text{ --- (5)}$$

$$(p + 12)x + (q + 12)y + (r + 12)z = f^2 \text{ --- (6)}$$

The above 6 equations have 12 unknowns.

Namely, (p,q,r,x,y,z,a,b,c,d,e,f)

We initially consider equation (1) & (2):

$$(p)x + (q)y + (r)z = a^2 \text{ --- (1)}$$

$$(p + 1)x + (q + 1)y + (r + 1)z = b^2 \text{ --- (2)}$$

We take $b=5a$ & $(p,q,r)=(1,2,1)$ & we get:

$$x + 2y + z = a^2 \text{ --- (7)}$$

$$x + y + z = 24a^2 \text{ --- (8)}$$

Solving equation (7) & (8) we get:

$$y = -23a^2 \text{ \& } (x + z) = 47a^2$$

we take, $9x = 10z$ & we get:

$$z = (423a^2)/19 \text{ --- (9)}$$

Inorder that 'z' be an integer we take, $a=19$ in equation (9)

$$\text{\& we get, } z = 8037 \text{ \& } x = \frac{10z}{9} = 8930, y = -23(19)^2 = -8303$$

Hence we have:

$$(x, y, z) = [(8930), (-8303), (8037)]$$

$$\text{\& since, } b=5a, b = 5(19) = 95$$

Now consider the below equation:

$$(p + n)x + (q + n)y + (r + n)z = w^2 \text{ --- (10)}$$

For $(p,q,r)=(1,2,1)$, equation (10) is equivalent to:

$$(x + 2y + z) + n(x + y + z) = w^2 \text{ --- (11)}$$

We know from equation (7) & (8), that, $(x+2y+z)=a^2$ & $(x+y+z)=24a^2$

Hence equation, (11) becomes:

$$w^2 = a^2(24n + 1) \text{ --- (12)}$$

Inorder for (LHS) of equation (12) to be a square we make $(24n+1)$ a square.

Now, $(24n+1)$ is a square at, $n=(2,5,7 \text{ \& } 12)$

\& we get: $w=[(7a),(11a),(13a),(17a)]$ for $n=(2,5,7 \text{ \& } 12)$

Since, $a=19$ & $n=(2,5,7 \text{ \& } 12)$, we have:

$$(c, d, e, f) = (= [(7a), (11a), (13a), (17a)] = [133,209,247,323]$$

Also, $(a,b)=(19,95)$ & since we have the below equation, we substitute in it

from equation (10) for, $n=(1,2,5,7,12)$ & we get:

$$(p)x + (q)y + (r)z = a^2 \text{ --- (1)}$$

$$(p + 1)x + (q + 1)y + (r + 1)z = b^2 \text{ --- (2)}$$

$$(p + 2)x + (q + 2)y + (r + 2)z = c^2 \text{ --- (3)}$$

$$(p + 5)x + (q + 5)y + (r + 5)z = d^2 \text{ --- (4)}$$

$$(p + 7)x + (q + 7)y + (r + 7)z = e^2 \text{ --- (5)}$$

$$(p + 12)x + (q + 12)y + (r + 12)z = f^2 \text{ --- (6)}$$

Thus we get after substituting the integer values for (a,b,c,d,e,f):

$$(p)x + (q)y + (r)z = (19)^2 \text{ --- (1)}$$

$$(p + 1)x + (q + 1)y + (r + 1)z = (95)^2 \text{ --- (2)}$$

$$(p + 2)x + (q + 2)y + (r + 2)z = (133)^2 \text{ --- (3)}$$

$$(p + 5)x + (q + 5)y + (r + 5)z = (209)^2 \text{ --- (4)}$$

$$(p + 7)x + (q + 7)y + (r + 7)z = (247)^2 \text{ --- (5)}$$

$$(p + 12)x + (q + 12)y + (r + 12)z = (323)^2 \text{ --- (6)}$$

In the above we have:

$$(p, q, r) = (1, 2, 1) \text{ \&}$$

$$(x, y, z) = [(8930), (-8303), (8037)]$$

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